

**PhD Qualifying Exam  
Nuclear Engineering Program**

**Part 1 – Core Courses**

**9:00 am – 12:00 noon, November 19, 2016**

### (1) Nuclear Reactor Analysis

During the startup of a one-region, homogeneous slab reactor of size ( $a$ ), placed in a vacuum, using the time-dependent, one-speed diffusion equation,

$$\frac{1}{v} \frac{\partial \phi}{\partial t} - D \frac{\partial^2 \phi}{\partial x^2} + \Sigma_a \phi(x, t) = \bar{v} \Sigma_f \phi(x, t)$$

we obtain the flux formulation expressed by

$$\phi(x, t) = \sum_{n \text{ odd}} A_n e^{-\lambda_n t} \cos(B_n x)$$

Where  $\lambda_n = v\Sigma_a + vDB_n^2 - v(\bar{v}\Sigma_f)$  and  $B_n = \frac{n\pi}{a}$ . Note that  $v$  is neutron speed, and  $\bar{v}$  is average number of neutrons per fission.

- If  $\lambda_1 > 0$ , show that the flux approaches the first harmonic (i.e.,  $n=1$ ) after a short time. (18%)
- What is the material buckling for this reactor? (4%)
- What is the criticality condition for this reactor? (7%)
- Show that the probability of non-leakage is expressed by  $P_{NL} = \frac{1}{1+B^2L^2}$  (21%)
- Show that the prompt neutron life-time is expressed by  $\ell_p = \frac{1}{v\Sigma_a(1+B^2L^2)}$  (18%)
- Show that  $\lambda_1 = \frac{(1-k)}{\ell_p}$  (25%)
- Using the expression in part (f), for a reactor with  $k = 1.003$ , using your guess for  $\ell_p$ , how much the neutron flux change in 1 sec? Is this a stable reactor? (explain your answers) (7%)

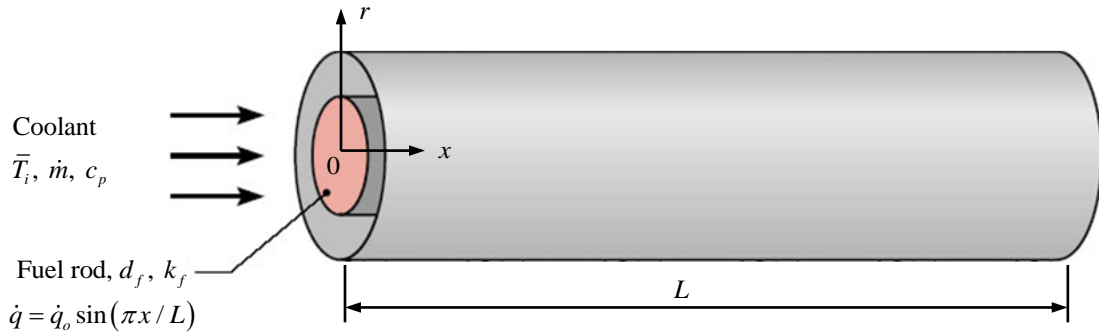
## (2) Reactor Thermal Hydraulics

Consider a cylindrical nuclear fuel rod of length  $L$  and diameter  $d_f$  that is encased in a concentric tube. Pressurized water flows through the annular region between the rod and the tube at a rate of  $\dot{m}$ , and the outer surface of the tube is well insulated. Heat generation occurs within the fuel rod, and the volumetric heat generation rate along the rod is given by:

$$\dot{q}(x) = \dot{q}_o \sin(\pi x / L),$$

where  $\dot{q}_o$  ( $\text{W}/\text{m}^3$ ) is a constant.

A uniform convection coefficient  $h$  may be assumed to exist between the surface of the rod and the water flow. Axial conduction can be neglected in rod and fluid. The specific heat of water  $c_p$ , and the thermal conductivity of the fuel rod  $k_f$  are constants. The system is in steady state condition.



- Obtain expressions for the local heat flux  $q''$  and the total heat transfer rate  $q$  from the fuel rod to the water. [15%]
- The mean temperature of the water at the inlet is  $\bar{T}_i$ . Obtain an expression for the variation of the mean temperature  $\bar{T}(x)$  of the water with distance  $x$  along the tube. [20%]
- Obtain an expression for the variation of the rod surface temperature  $T_s(x)$  with distance  $x$  along the tube. Develop an expression for the  $x$  location at which this temperature is maximized. [30%]
- Obtain an expression for the variation of the temperature inside the fuel rod  $T_f(x, r)$  with distance  $x$  along the tube and radial distance  $r$  from the centerline. Develop an expression for the location  $(x, r)$  at which  $T_f(x, r)$  is maximized. Assume the temperature distribution is axisymmetric in the fuel rod, and the axial heat conduction can be neglected. [35%]

Hint: the Laplacian operator in the cylindrical coordinate system can be given as:

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial x^2}$$

### **(3) Advanced Nuclear Materials**

Answer the following questions:

- a) Describe thermal spikes mechanism? (20%).
- b) What is the effect of neutron radiation on the material yield strength? Describe the mechanism. (20%)
- c) Where does He atom come from in fuel cladding? (20%)
- d) What is embrittlement? Where does this problem come from? (20%).
- e) Describe the threshold energy required to displace an atom under neutron irradiation? (20%).

#### **(4) Radiation Detection and Shielding**

##### Half-life of a Long-lived Radionuclide:

The following experiment was then conducted to measure the K-40 half-life.  $300.0 \pm 0.1$  mg of reagent-grade potassium chloride (*KCl*) was measured out in a 2.5 cm diameter planchette. The planchette was placed 1 cm away from the window of a Geiger-Mueller detector. The Geiger-Mueller detector window has a diameter of 3 cm. The effective solid angle averaged over the surface of the sample presented to the detector is 2.336 steradians.

The sample and G-M counter are located inside a container shielded by 4-cm of lead. A 30-minute background count rate is determined before and after counting the sample. The two background counts are averaged. For this experiment, the average background count rate was  $3.5 \pm 0.3$  cpm. The *KCl* sample was then counted for four hours obtaining a total of 11,621 counts.

The following data is provided:

Abundance of  $^{40}\text{K}$  in natural potassium,  $\gamma_{40}$ :  $1.17 \times 10^{-4}$ . The gram-atomic weight of  $^{40}\text{K}$  is 39.9640 g/mole. The gram-molecular weight of *KCl* is 74.5513 g/mole.

$^{40}\text{K}$  undergoes beta-minus decay 89.28% of the time with an average beta energy of 560.18 keV and max beta energy of 1311.07 keV. It also undergoes electron capture decay 10.67% of the time releasing a gamma-ray of energy 1460.82 keV.

The Geiger-Mueller detector has an efficiency of counting beta particles at 98%. Its efficiency for counting gamma-rays at 1460 keV, however, is essentially zero. Loss due to absorption of soft beta-rays in the mica window of the Geiger-Mueller detector is estimated to be 5%.

- a) Using the provided information calculate the half-life of  $^{40}\text{K}$  in years. (40%)
- b) Assuming the only uncertainties are the background count rate, the total sample count, and the measurement of the *KCl* mass, estimate the expected standard deviation of the half-life value in years. (60%)

**(5) Advanced Engineering Mathematics**

The Maxwellian density function expressed by

$$M(E, T) = 4\pi \left( \frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

Where,  $m$  is mass,  $k$  is the Boltzmann constant,  $T$  is temperature in Kelvin, and  $v$  is speed.

- a) Derive the density function in terms of kinetic energy (E) (33%)
- b) Show that the most probable energy does not correspond to the most probable speed (38%)
- c) Determine the average energy (29%)

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**Part 2 – Specialty Courses  
Radiation Detection & Shielding**

**2:00 pm – 5:00 pm, November 19, 2016**

## **Specialty - Radiation Detection & Shielding (1) (37.5%)**

### Transient Equilibrium

When the half-life of the parent is slightly greater than that of the daughter, transient equilibrium can occur. In this situation, the daughter activity builds up to a maximum and then decays away.

- a) For the case where you have the decay chain  $X_1 \rightarrow X_2 \rightarrow X_3$  where  $X_3$  is stable, starting out with 100% of the parent  $X_1$  with an initial activity  $A_{1,0}$ , derive an equation for the total radioactivity of the parent plus the daughter with time, i.e.  $A_{\text{tot}}(t) = A_1(t) + A_2(t)$ . (65%)
- b) Derive an expression for the time at which the total activity derived above reaches a maximum. (35%)



## **Specialty - Radiation Detection & Shielding (2) (37.5%)**

A 1-Ci point source of  $^{24}\text{Na}$  is to be stored in a lead box. The radionuclide emits two photons per disintegration with energies of 2.75 MeV and 1.37 MeV in decaying by  $\beta^-$  emission to stable  $^{24}\text{Mg}$ . How thick must the lead walls on the box be such that the exposure rate 1 meter away from the 1-Ci point source is less than 2 mR/hr?

The exposure rate,  $\dot{X}$ , can be calculated by:

$$\dot{X} = \Gamma \frac{\alpha}{d^2},$$

where  $\alpha$  is the activity in milliCuries of the point source,  $d$  is the distance in centimeters from the point source, and  $\Gamma$  is exposure rate constant from the Table below.

Density of lead = 11.4 g/cm<sup>3</sup>

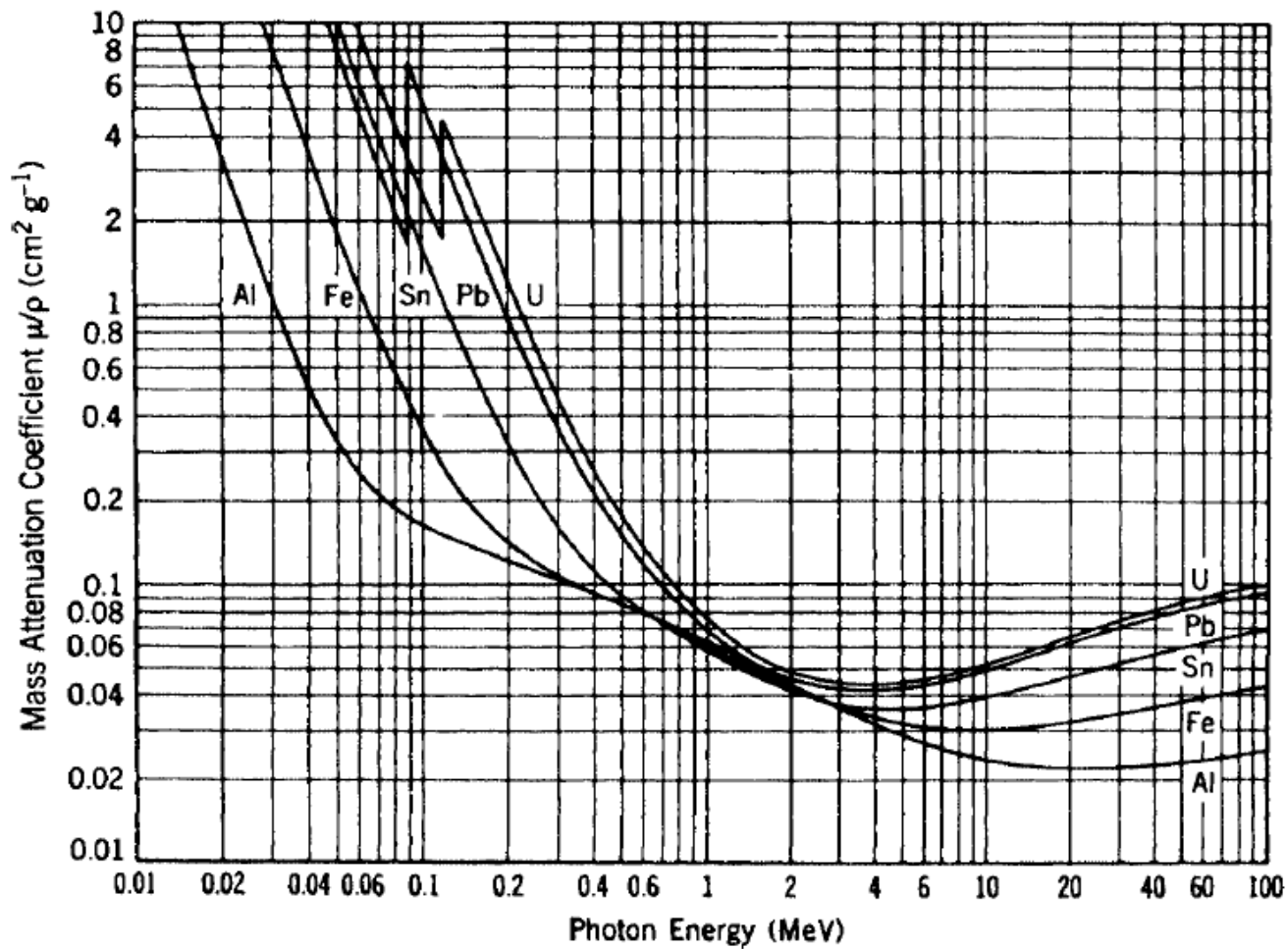
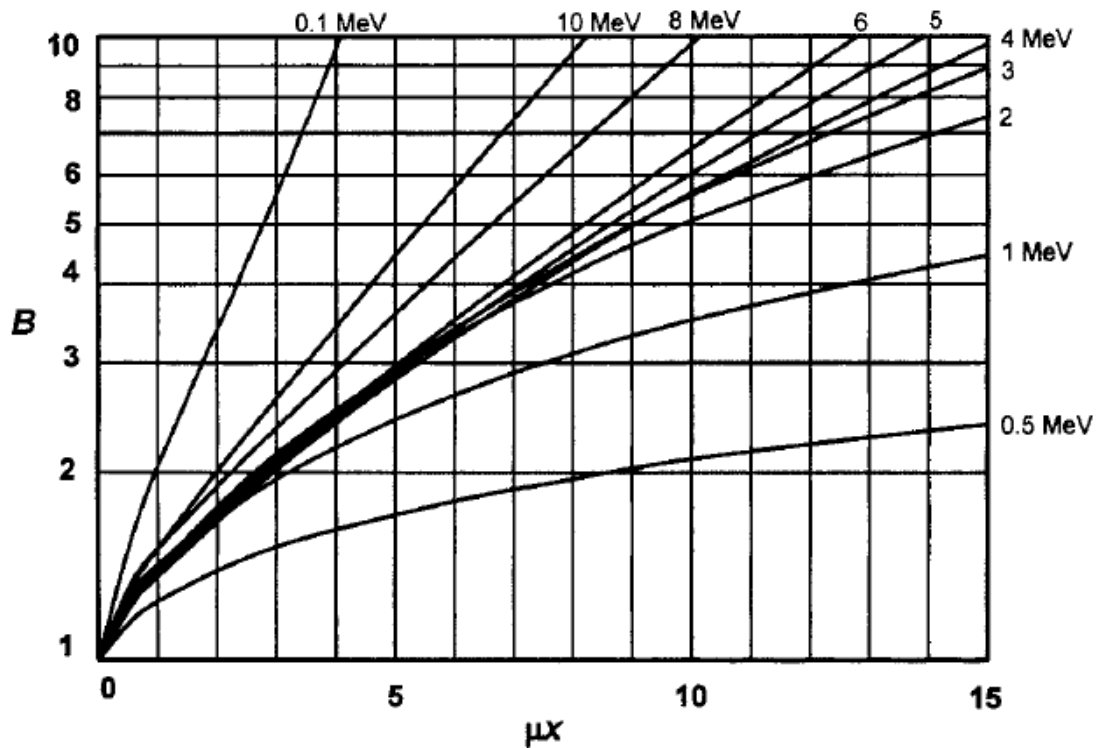
**Table 2.1** Exposure Rate Constant for Some Common Radioisotope Gamma-Ray Sources

Nuclide	$\Gamma^a$
Antimony-124	9.8
Cesium-137	3.3
Cobalt-57	0.9
Cobalt-60	13.2
Iodine-125	~0.7
Iodine-131	2.2
Manganese-54	4.7
Radium-226	8.25
Sodium-22	12.0
Sodium-24	18.4
Technetium-99m	1.2
Zinc-65	2.7

<sup>a</sup>The exposure rate constant  $\Gamma$  is in units of (R · cm<sup>2</sup>)/(hr · mCi).

See next page

# LEAD



## **Specialty - Radiation Detection & Shielding (3) (25%)**

### **Radiation Detector Operation**

Discuss the following topics/concepts related to a radiation detector:

- a) Draw the gas multiplication curve as a function of pulse height versus applied voltage. (15%)
- b) Using the different regions of the gas multiplication curve above explain the operation of applicable gas-filled detectors and how these detectors take advantage of the features of this curve to work. Include a detailed description of these detectors. (50%)
- c) Indicate how the gas multiplication curve in part (a) above would change for an incoming alpha particle versus the curve for an incoming beta particle. Explain why the curve changes the way it does. (10%)
- d) Explain how a semiconductor diode detector works to detect radiation. In your explanation, include the reason reverse bias is used. (25%)

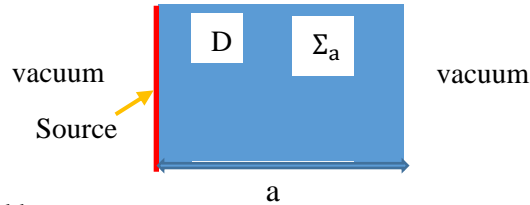
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**Part 2 – Specialty Courses  
Particle Transport Methods and Reactor Physics**

**2:00 pm – 5:00 pm, November 19, 2016**

**Specialty - Particle Transport Methods and Reactor Physics (1) (25%)**

A one-region slab of size  $a$  is placed in a vacuum. An anisotropic planar source is placed on the left boundary of the slab as depicted below



The angular source is expressed by

$$S(\hat{\Omega}) = S_0 \left( \frac{1}{4\pi} + \frac{1}{8\pi} \mu \right)$$

Where,  $\hat{\Omega}$  is unit vector particle direction, and  $\mu$  is direction cosine of polar angle corresponding to direction  $\hat{\Omega}$ .

- Derive a formulation for neutron flux distribution in the slab using the one-speed diffusion equation.

Hint: The one-speed diffusion equation is given by

$$-D \frac{d^2 \phi}{dx^2} + \Sigma_a \phi(x) = S(x),$$

and partial current formulations based on the diffusion approximation are expressed by

$$J_{\pm}(x) = \frac{\phi(x)}{4} \mp \frac{D}{2} \frac{d\phi(x)}{dx}$$

**Specialty – Particle Transport Methods and Reactor Physics (2) (37.5%)**

The one speed, 1-D transport equation in the Cartesian geometry is given by

$$\mu \frac{\partial \psi(x, \mu)}{\partial x} + \sigma_t \psi(x, \mu) = 2\pi \int_{-1}^1 d\mu' \sigma_s(x, \mu_0) \psi(x, \mu') + S(x, \mu),$$

- a) Derive expressions of the scattering and source terms if both scattering and source are isotropic (23%)
- b) Derive the one-speed, 1-D discrete ordinates ( $S_N$ ) equations (Do not need to perform spatial discretization) (13%)
- c) Discuss a methodology for determining the  $S_N$  quadrature set in a 1-D geometry (37%)
- d) To derive the  $P_L$  equations, briefly explain the procedure/formulation you would use (Do NOT derive the  $P_L$  equations.) (13%)
- e) Under what conditions the  $P_L$  and  $S_N$  equations are equivalent? (7%)
- f) What is the difficulty with the  $P_L$  equations? (7%)

**Specialty- Particle Transport Methods and Reactor Physics (3) (37.5%)**

The Monte Carlo method is used for solving an integral such as

$$I = \int_a^b dx f(x)g(x)$$

- a) Explain the standard procedure and formulation used to solve the above integral (17%)
- b) Briefly explain the importance sampling method for solving an integral (10%)
- c) Derive the theoretical variance corresponding to the following integral (17%)

$$\int_0^1 dx(1 - x^2)$$

- d) Using the importance sampling approach, propose an effective  $pdf(f^*(x))$  to solve the integral in part c, and derive the corresponding variance (40%)
- e) Compare the results of the parts c and d (3%)
- f) Name two other variance reduction techniques for solving the above integral; which component of the integral is modified in these techniques? (Do not solve the integral with these techniques) (13%)